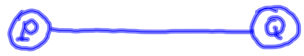
WSPD



$d(P,Q) \ge \frac{1}{\varepsilon}\max(\mathrm{diam}(P),\mathrm{diam}(Q))$

$n=|P|$

$P\subseteq\mathbb{R}^d$ $\longrightarrow$ $\frac{1}{\varepsilon}$-WSPD $O(\frac{n}{\varepsilon^d})$ pairs

$\varepsilon>0$

$O(n\log n + \frac{n}{\varepsilon^d})$

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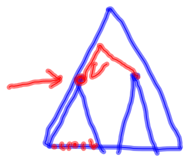
$P \to CQT(P)$

$rep_v \in P_v$ $\quad$ rep_v $\to$



t-spanner

$G=(V,E)$ weighted

$H=(V,E')$ is a t-spanner iff

$\forall u,v\in V$ $\quad d_G(u,v)\le d_H(u,v)\le t\cdot d_G(u,v)$.

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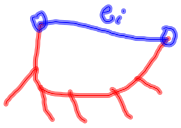
$e_1, e_2, \dots, e_m$ – sorted



$O(n^{1+\overset{2}{\downarrow}1/t})$

Spanners from WSPD = $(1+\varepsilon)$-spanner.

$P\subseteq\mathbb{R}^d$ $\quad$ $W \equiv \frac{4}{\varepsilon}$-WSPD(P) $\quad$ $O(n\log n + \frac{n}{\varepsilon^d})$

edges $O(\frac{n}{\varepsilon^d})$ $\quad$ $t=1+\varepsilon$

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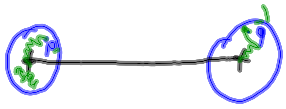
"Proof"

Induction on distances.

$\binom{P}{2} = \{\underbrace{\{p,q\}, \{p_2,q_2\}, \dots}_{k \text{ pairs}}\}$

$\frac{4}{\varepsilon}$-WSPD



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$(1+\varepsilon)$-MST

$P\subseteq\mathbb{R}^d$ – MST(P)

$\varepsilon>0$ – approx param

$(1+\varepsilon)$-spanner $\quad G=(P,E)$

$T=MST(G)$ $\qquad$ Exact MST

$O(n\log n+\frac{n}{\varepsilon^d})$ $\quad\neq\quad$ $O(n^{2-\frac{1}{d}})$

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Diameter

$P\subseteq\mathbb{R}^d$

$p,q\in P$ $\quad \|p-q\|\ge(1-\varepsilon)\,\mathrm{diameter}(P)$

Alg $\quad \frac{4}{\varepsilon}$-WSPD $\quad O(n\log n + \frac{n}{\varepsilon^d})$

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All nearest neighbors

$P \subseteq \mathbb{R}^d$

$\forall p \in P$ compute it's nearest neighbor in $P \setminus \{p\}$

Thm ANN can be solved in $O(n \log n)$ time.

Lemma ANN can be " in $O(n(\log n + \log \Phi))$

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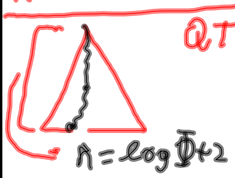
All NN bounded spread



$QT(P)$

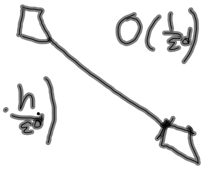
$O(n \log \Phi + \frac{n}{\varepsilon^d})$

$\frac{1}{\varepsilon}$-WSPD

$\{A_1, B_1\}, \ldots, \{A_m, B_m\}$

$\sum_i (|A_i| + |B_i|) = O(n \cdot \frac{h}{\varepsilon^d})$

$O(\frac{h}{\varepsilon^d})$



$O(\frac{1}{\varepsilon^d})$

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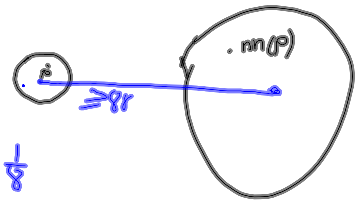

$\varepsilon = \frac{1}{8}$

$\{\{q\}, \{s_1, \ldots, s_8\}\}$

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Shifting- Random

HST

Hierar' spanning tree

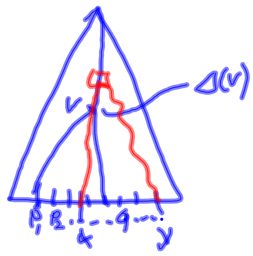
(X,d) - metric

H - tree, labels

$\forall x,y$

$d(x,y) \leq \Delta(lca(x,y))$

$\leq t \cdot d(x,y)$



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PHST

Build HST s.t.

$d(x,y) \leq E[\Delta(x,y)] \leq O(d(x,y) \log n)$

Bartal / FRT

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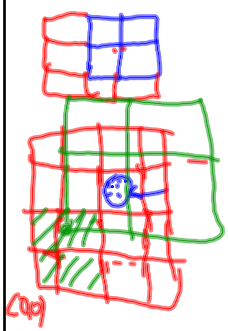
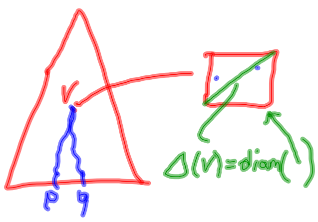
CQT(P)





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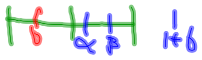
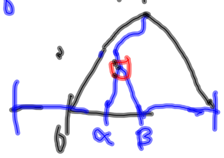
$h_{b,\Delta}(x) = \left\lfloor \frac{x-b}{\Delta} \right\rfloor$ $\left\lfloor \frac{x}{\Delta} \right\rfloor$

$L_b(\alpha,\beta) = \underbrace{1 - \text{bit}(\alpha - b,\ \beta - b)}_{\text{level of the lca of } \alpha,\ \beta}$





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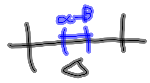
Lemma

$\Pr\left[L_b(\alpha,\beta) > \lg|\alpha-\beta| + t\right] \le \frac{4}{2^t}$



$1 - \frac{|\alpha-\beta|}{\Delta}$

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$L_b(p,q) = \max_{i=1}^{d} L_{b_i}(p_i, q_i)$

$p = (p_1, \dots, p_d)$

$b = (b_1, \dots, b_d)$

Lemma $P \subseteq \left[\frac{1}{2}, \frac{3}{4}\right]^d$

with prob $\ge 1 - \frac{4d}{n^{c-2}}$ we have

$\forall p,q \in P \quad L_b(p,q) \le \lg\|p-q\| + c\lg n$

$\Rightarrow$ Rand shifted QT is n^c-HST.

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